



Generic model identification framework for thermodynamic engines for use in hybrid power stations control and simulation



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ABSTRACT

Thermodynamic engines are focusing increasing attention in the context of solar-based electric power generation. Knowledge-based models of such engines are sometimes difficult to derive and when they are available, their simulation may be numerically a rather heavy task given the control updating period that may be needed. In the present work a generic nonlinear identification framework that enables the dynamics of the key quantities of a thermodynamic engine to be captured is proposed. Such a fast model can then be used in the simulation and the control design stage of the whole electric power generation station. The proposed identification framework is validated on a recently developed knowledge-based model of a beta-type Stirling engine with rhomic-drive mechanism.

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1. Introduction

Thermodynamic engines transform the thermal energy into mechanical energy. This is done through a positive energy cycle in which a fluid circulates between a hot source and a cold source. The resulting mechanical energy can therefore be used to drive an alternator which is followed by a voltage conditioning stage leading to a usable electrical energy by the end user loads.

A typical view of a thermodynamic engine in such a global hybrid power station framework is shown in Fig. 1.

Running such hybrid power stations is today an important challenge as it can be attested by the recent works involving the use of renewable wind/photovoltaic related energy sources [7,1,10,11]. One reason is that more than 1.4 billion people have no access to electricity, mainly in Asia and Africa. Classical alternative fuel-based solutions such as diesel engines suffer from many drawbacks such as their low reactivity to load changes, their high emissions and noise to mention only few issues [8].

Roughly speaking, a thermodynamic engine in a hybrid power generation station provides the medium term power adaptation while an electric storage element (super-capacitor and/or battery)

accommodates for sudden changes in the load power demand. It results (see [4] for a detailed demonstration) that the dynamics of the thermodynamic engine highly determine the size and the type of the storage element. It determines also to which extent one can avoid the use of batteries which is becoming nowadays necessary for obvious ecological reasons.

Now in order to perform such a closed-loop quantitative analysis on a variety of scenario, it is necessary to dispose of a fast simulator of the thermodynamic engine in its loop. This can be obtained either using a knowledge-based model or using a black-box model. Note also that even when a knowledge-based model is available, it can be used to derive black-box model in order to accelerate the simulation and the control algorithm development task.

To the authors knowledge, several companies are currently developing thermodynamic engines following the basic concepts of positive energy cycles (Stirling, Rankine, etc.). Based on some elementary computation schemes, the main parameters of the engine can be computed given the targeted output mechanical power range. It comes out that thermodynamic engines can be available before a dynamic model is derived. In this case, real-life experiments can be conducted to derive black-box models without the need for a mathematical knowledge-based model. Note however that knowledge-based models are mandatory for process optimization in dynamic context. Black-box models can only reproduce a predesigned process behavior.

The aim of this paper is to give a generic easily identifiable nonlinear structure that captures the dynamics of the key quantities of a thermodynamic engine which are necessary to simulate it in its

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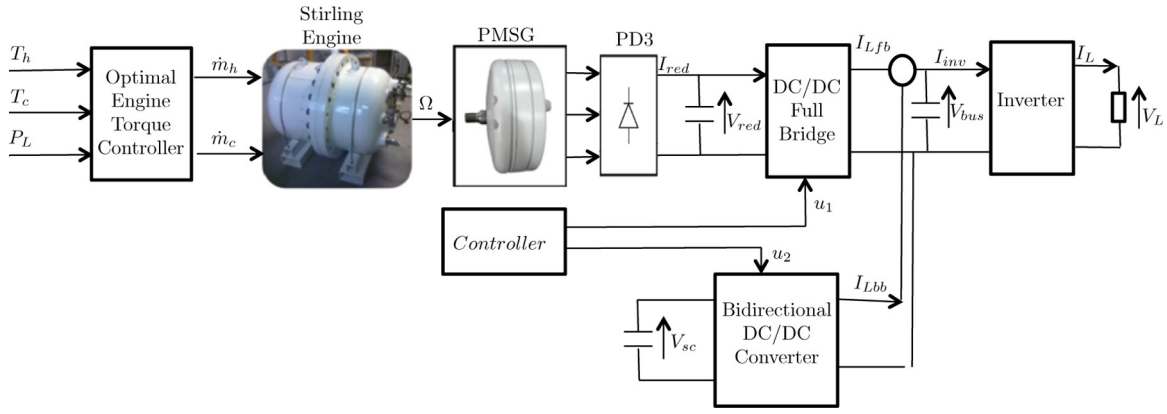


Fig. 1. An example of a hybrid power station with its thermodynamic engine (here a beta-type Stirling engine) followed by the electric power conditioning stage.

environment. The adopted structure has been recently proposed in [2,3] where it has been used to derive a fast moving-horizon state observer [2] and to derive a dynamic model of NOx emission in a diesel engine [3].

In the absence of experimental data, the proposed identification scheme is validated in the present contribution using the very detailed model of a beta-type Stirling engine recently proposed in [5]. This knowledge-based model takes into account, among other non-ideal features, the non-isothermal effects, the effectiveness of the regenerative channel, and the thermal resistance of the heating head. This is why such a detailed model can reasonably emulate an experimental data generator for the identification method proposed in this paper. This holds as far as only the presumably measured signals are used to build the black-box model.

It is worth underlying that although the above cited specific Stirling engine is used throughout the paper, the identification structure and methodology are quite general and should be successfully used to derive a databased model for any thermodynamic engine. This is because almost all thermodynamic engines involve the same components and describe very similar cycles in the (P, V) coordinates. For instance, the framework proposed in the present paper has been also successful in identifying a Rankine-cycle-based engine. However, the corresponding results cannot be published for industrial reasons.

The contributions of the papers can be summarized by the following items:

- (1) The paper suggests a systematic parsimonious identification problem setting that targets only those variables that are relevant for power stations simulation and control. In the specific case of the Stirling engine considered in the present paper, these variables are the mean work and the mean heat exchange during an engine cycle. Note that this problem definition is independent of the method and the identification structure that can be used to solve the resulting identification problem.
- (2) The second contribution of the paper is to show that using a recently developed identification framework, the identification problem described in the preceding item leads to astonishingly faithful models of the dynamics of the targeted key quantities (work and heat exchange) that need to be captured. It is also conjectured (for the reasons cited above) that this success is likely to be encountered on a whole family of thermodynamic engines that are currently widely developed for green energy generation and conversion.

It is by no means the claim of the present paper that the identification framework used herein is the only one capable of producing

nice matching results. It is true however that the proposed framework does show nice properties that are recalled in Section 3.

The paper is organized as follows:

- In Section 2, the identification problem that has to be solved in order to derive a parsimonious dynamic model of the thermodynamic engine is stated. By *parsimonious*, it is meant a dynamic model that incorporates only the necessary elements that are needed to interact with the remaining blocks of the hybrid power generation station (see Fig. 1) and to design the feedback laws.
- In Section 3, the general identification scheme proposed in [2,3] is briefly recalled and its application in the specific identification context described in Section 2 is discussed. It is then shown that the identification problem can be put into a general nonlinear static map identification.
- Section 4 gives a brief sketch of the knowledge-based model of the beta-type Stirling engine with rhomic-drive mechanism that has been recently proposed in [5] and used in the next section as a data generator for the identification task.
- The identification framework is validated in Section 5 where the learning/validation data are described and the identification results are shown.
- Finally, Section 6 concludes the paper and give hints for further investigations.

2. The identification problem

In this section, the necessary elements that need to be identified regarding the thermodynamic engine in order to build the simulator of the whole hybrid power generation station are analyzed. This enables the identification problem to be clearly stated.

The hybrid power station model can be shortly written as follows:

$$I\dot{\Omega} = \Gamma_e(\Omega, x^e, u^e) + \Gamma_m(\Omega, x^{th}, u^{th}, w^{th}) - \Gamma_f(\dots) \quad (1)$$

$$\dot{x}^e = F(x^e, \Omega, u^e, w^e) \quad (2)$$

where Ω (rad/s) is the angular velocity of the main shaft that is shared by the engine and the alternator. x^e and u^e are the state and the control vectors of the electrical part. x^{th} and u^{th} are the state and the control vectors of the thermodynamic engine. Γ_e is the torque applied on the shaft from the electric part while Γ_m is the mechanical torque that results from the Stirling engine positive energy cycle. Γ_f is the friction torque. Finally, the vectors w^e and w^{th} represent the non controlled signals acting on the electrical part (user power, disturbances) and the thermodynamic part (hot temperature T_H for instance). For the complete definition of F , interested readers can refer to the description given in [4].

A typical instantiation of the electrical equation (2) can be found in [4] where a voltage conditioning stage is used including a super-capacitor. In the present paper, this part of the power generation station is not considered since the interest is focused on the derivation of a fast and parsimonious model of the thermodynamic engine.

It comes out that the rotational speed Ω is the coupling quantity between the mechanical and the electrical parts and that the only signal that summarizes the behavior of the thermodynamic engine is the torque Γ_m that can be linked to the power P_m delivered by the engine through the following equation:

$$\Gamma_m(\Omega, x^{th}, u^{th}, w^{th}) = \frac{P_m(\Omega, x^{th}, u^{th}, w^{th})}{\Omega} \quad (3)$$

Now given the mechanical inertia, the mean power $\bar{P}_m$ during a cycle can be used instead of the instantaneous power. But the mean power $\bar{P}_m$ can be given by:

$$\bar{P}_m = \frac{W_{out}}{T}; \quad T := \frac{2\pi}{\Omega} \quad (4)$$

where W_{out} is the mechanical work produced during a single thermodynamic cycle. This leads to:

$$\Gamma_m \approx \frac{1}{2\pi} W_{out}(\bar{\Omega}, \bar{x}^{th}, \bar{u}^{th}, \bar{w}^{th}) \quad (5)$$

where the arguments are the mean values of the original arguments over the current cycle.

This means that as far as simulation of the whole system is concerned, the key thermodynamic quantity to be identified is the work produced by the thermodynamic engine during a cycle for a given mean rotational speed, given mean state and given mean input vectors.

As mentioned before, one of the reasons for which a fast dynamic model is needed lies in the control design issue. Indeed, it is worth operating the engine in a high efficiency region in the (power, speed) map. This map generally shows a bell-like curve that may depend on the input u^{th} for a given rotational speed Ω .

Therefore, beside the identification of the map W_{out} involved in (5) one needs to identify the efficiency map:

$$\eta = \frac{W_{out}(\bar{\Omega}, \bar{x}^{th}, \bar{u}^{th}, \bar{w}^{th})}{Q_{in}(\bar{\Omega}, \bar{x}^{th}, \bar{u}^{th}, \bar{w}^{th})} \quad (6)$$

where Q_{in} is the heat exchange with the hot source during a cycle.

The expressions of W_{out} and Q_{in} in terms of the measured quantities depend on the specific engine at hand. In Section 4, the expressions of W_{out} and Q_{in} (and hence the efficiency η) are given for the specific case of the beta-type Stirling engine with rhomic-drive mechanism.

2.1. Notation

In the remainder of this section and in order to simplify the expressions, the vectors u is temporarily used to denote both the rotational speed, the controlled and the exogenous signal of the thermodynamic engine, namely:

$$u := \begin{pmatrix} \Omega \\ u^{th} \\ w^{th} \end{pmatrix} \quad (7)$$

The rationale behind this is that from the identification point of view, there is no need to distinguish these inputs. Note that during the identification experiment, the rotational speed can be imposed using controlled motor as a load and can therefore be viewed as exogenous signal. Moreover, the superscript th is omitted since only the thermodynamic engine's variables are concerned in the sequel.

Therefore, the two quantities to be identified are given by:

$$W_{out}(\bar{x}, \bar{u}); \quad Q_{in}(\bar{x}, \bar{u}) \quad (8)$$

since then $\eta(\bar{x}, \bar{u})$ is obtained through (6). Now since the two quantities represent the produced mechanical work and the heat exchange evaluated on a cycle, cycle-related notation is necessary to properly express the identification problem. These are given below.

Note first of all that a cycle duration is defined to be the time interval between two successive instants t_k and t_{k+1} such that:

$$\theta(t_k) \in \{2i\pi \mid i \in \mathbb{N}\}; \quad \theta(t_{k+1}) = \theta(t_k) + 2\pi \quad (9)$$

where θ is the angle of the Stirling engine shaft.

Let us denote by $t_k^{(i)}$ the successive measurement acquisition instants inside the time interval $[t_k, t_{k+1}]$, namely:

$$t_k \leq t_k^{(i)} < t_k^{(i+1)} < \dots < t_k^{(N_k^m)} \leq t_{k+1} \quad (10)$$

Note that the number N_k^m of such instants does depend on k since the length of a cycle is not constant and depends on the rotational speed profile $\Omega(\cdot)$ around the instant t_k . More precisely:

$$2\pi = \int_{t_k}^{t_{k+1}} \Omega(t) dt \quad (11)$$

Given a cycle number k , the following notation:

$$x(t, k); \quad u(t, k); \quad \Omega(t, k); \quad t \in [t_k, t_{k+1}] \quad (12)$$

is used hereafter to refer to the value of the state, control and rotational speed over the cycle duration.

The mechanical work and the heat exchange during the cycle number k are denoted by $W_{out}(k)$ and $Q_{in}(k)$. One obviously has:

$$W_{out}(k) = \mathcal{W}(x(t_k, k), u(\cdot, k)); \quad Q_{in}(k) = \mathcal{Q}(x(t_k, k), u(\cdot, k)) \quad (13)$$

since everything that happens during a cycle depends on the initial state $x(t_k, k)$ at the beginning of the cycle and the input profile $u(\cdot, k)$ during the cycle.

2.2. Removing the state vector

One of the obviously difficult issues in identifying the maps (8) lies in the presence of the state vector x . Indeed, if the entire state vector is used in the identified map, then the use of the black-box model would need the whole state vector to be reconstructed which may be cumbersome. Fortunately, this difficulty can be overcome thanks to the inherent open-loop stability of the thermodynamic engines.

Indeed, open-loop stability of cyclic processes implies that the effects of initial state disappear asymptotically as time progresses. Using the notation above, this property can be formally written as follows:

$$x(t_k, k) = G(u(\cdot, k - N + 1), \dots, u(\cdot, k - 1)) + O\left(\frac{1}{N}\right) \quad (14)$$

which can be read informally as follows:

The value of the state at the beginning of a cycle can be approximately viewed as a function of the input profiles during the last N cycles provided that N is sufficiently high.

Now using (13) together with (14) enables to write

$$W_{out}(k) \approx \mathcal{W}(G(u(\cdot, k - N + 1), \dots, u(\cdot, k - 1)), u(\cdot, k)) \quad (15)$$

$$Q_{in}(k) \approx \mathcal{Q}(G(u(\cdot, k - N + 1), \dots, u(\cdot, k - 1)), u(\cdot, k)) \quad (16)$$

Now assuming that the variations of the components of the input u are slow when compared to the duration of the cycle, one can

replace $u(\cdot, k)$ by the mean value $\bar{u}(k)$ during the period, this leads to:

$$W_{out}(k) \approx \mathcal{W}(G(\bar{u}(k - N + 1), \dots, \bar{u}(k - 1)), \bar{u}(k)) \quad (17)$$

$$Q_{in}(k) \approx \mathcal{Q}(G(\bar{u}(k - N + 1), \dots, \bar{u}(k - 1)), \bar{u}(k)) \quad (18)$$

which can be written in a more compact form using obvious definitions of F_1 and F_2 :

$$W_{out}(k) = F_1(Z(k)); \quad Q_{in}(k) = F_2(Z(k)) \quad (19)$$

where $Z(k)$ is defined by:

$$Z(k) := \begin{pmatrix} \bar{u}(k) \\ \vdots \\ \bar{u}(k - N + 1) \end{pmatrix} \in \mathbb{R}^{n_z}; \quad n_z := N \cdot n_u \quad (20)$$

In the sequel, the vector Z is referred to as the *regressor*. Anticipating on the presentation of Sections 4 and 5, the components of the vector u involved in the definition of the regressor Z are given by:

$$u := \begin{pmatrix} \Omega \\ T_H \\ \delta \end{pmatrix} \quad (21)$$

where T_H (K) is the temperature of the hot source and δ is a parameter that controls the flow rate of the air in the regenerator. Obviously, one can add also the cold temperature T_L but this is considered here to be constant as it does not depend on the sunlight.

In the following section, a recently proposed general framework [3] for the identification of nonlinear maps such as F_1 and F_2 is recalled.

3. Identification of general nonlinear relationships

The identification of the two maps F_1 and F_2 involved in (19) gathers two instantiations of the following general nonlinear identification problem:

Definition 1 (*The Identification Problem*). Given the data:

$$\{(q(k), Z(k))\}_{k=1}^{N_{data}} \quad (22)$$

find a map $F: \mathbb{R}^{n_z} \rightarrow \mathbb{R}$ that approximately maps Z to q in a least squares sense, that is:

$$q \approx F(Z) \quad (23)$$

Identification of nonlinear relationships is an open issue. Many frameworks have been proposed including neural networks, Wiener–Hammerstein, Volterra series based formulations [9,6] to cite but few possibilities.

Although any of these methods can be used to solve the identification problem at hand, the method used in the sequel is the one that has been recently proposed in [3] since it shows the advantage of being based on the solution of a Quadratic Programming (QP) problem. The consequences of this feature are the following:

- (1) Since the solution of QP problems can be done extremely fast, it is possible to perform on-line updating of the model during the system life-time.
- (2) QP problem solution does not suffer from possible local minima. This feature can be used in an outer-loop in which the parameters of the identification structure can be optimized as it is explained later in this section.

The price to pay in order to put the problem in a QP form is to restrict the class of functions F to the one given by:

$$F(Z) = \Gamma^{-1}(Z^T L); \quad \Gamma(\cdot) \text{ strictly increasing} \quad (24)$$

Note that since Γ is strictly monotonic, Γ^{-1} does exist. The rationale behind this restriction is that the original problem of finding F can be replaced by the one consisting in finding $L \in \mathbb{R}^{n_z}$ and Γ such that:

$$\Gamma(q) \approx Z^T L \quad (25)$$

Now assume that the map Γ is parametrized using the following structure:

$$\Gamma(q) = \sum_{j=1}^{n_b} [B^{(j)}(\xi(q))] \cdot \mu_j; \quad \xi(q) := \frac{q - q_{\min}}{q_{\max} - q_{\min}} \quad (26)$$

in which $q_{\min}$ and $q_{\max}$ are the lower and the upper values of q over the identification data (22) while $B^{(j)}(\cdot)$ are a set of functions that are defined on $[0, 1]$.

The detailed analytical definition of the maps $B^{(j)}(\cdot)$ is given in Appendix A.1. Fig. 2 shows the allure of these maps for $\beta = 0.5$.

Remark 1. The choice of the function basis shown in Fig. 2 is not unique. Nevertheless, this choice is motivated by the monotonicity assumption on Γ . Indeed, any one of the curves shown in Fig. 2 is a monotonic function that can be viewed alone as a candidate function for Γ while each one of them would need many terms of standard basis functions such as polynomials to be correctly represented.

Note that the expression of $\Gamma(q)$ can be put in a matrix form using the vector of parameter $\mu \in \mathbb{R}^{n_b}$ and an obvious definition of $B(q) \in \mathbb{R}^{1 \times n_b}$ as follows:

$$\Gamma(q) = [B(\xi(q))] \cdot \mu \quad (27)$$

which enables (25) to be transformed into:

$$[B(\xi(q)), -Z^T] \cdot \begin{pmatrix} \mu \\ L \end{pmatrix} \approx 0 \quad (28)$$

that has to be solved in least squares sense under the following two linear constraints on the vector of coefficients μ

- The first constraint expresses the fact that Γ has to be strictly monotonic:

$$\forall \xi \in [0, 1], \quad \left[\frac{dB}{d\xi}(\xi) \right] \cdot \mu > \epsilon \quad (29)$$

- The second constraint is a normalization constraint that avoids the trivial solution $\mu = 0, L = 0$:

$$\left[\int_0^1 B(\xi) d\xi \right] \cdot \mu = \frac{q_{\min} + q_{\max}}{2} \quad (30)$$

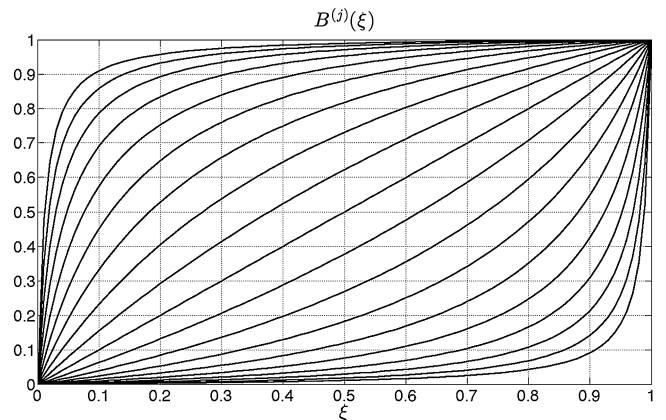


Fig. 2. Allure of the maps $B^{(j)}(\cdot)$ for $\beta = 0.5$ and $n_m = 10$.

Note that (29) and (30) can be transformed into a finite number of linear constraints by defining a sufficiently dense grid of values of ξ over the interval $[0, 1]$.

To summarize, the two optimal parameters of the solution, namely $\hat{L} \in \mathbb{R}^{n_z}$ and $\hat{\mu} \in \mathbb{R}^{n_b}$ are obtained by solving the constrained linear least squares defined by (28)–(30).

Note that the degree of nonlinearity of the solution can be evaluated by the indicator:

$$\gamma_{nl} := \frac{\max_{\xi \in [0,1]} |(dB)/(d\xi)(\xi)| - \min_{\xi \in [0,1]} |(dB)/(d\xi)(\xi)|}{\max_{\xi \in [0,1]} |(dB)/(d\xi)(\xi)|} \quad (31)$$

which is obviously a number that belongs to $[0, 1]$ and that equals 0 for linear models. Note that the denominator in (31) is obviously non vanishing thanks to the monotonicity constraint (29).

Remark 2. Regarding the choice of the parameters N and n_b involved respectively in the definition of the regressor size and the number of functions used to represent the map Γ , the following procedure can be used:

- (1) Choose maximum values $n_m^{\max}$ and $N^{\max}$ on these parameters.
- (2) Solve the QP problem on the grid of possible values defined by the set:

$$\{1, \dots, n_m^{\max}\} \times \{1, \dots, N^{\max}\}$$

This can be done extremely rapidly since the optimization problem is a simple QP.

- (3) Denote by $r_{\min}$ the value of the lowest residual which is typically obtained for the pair $(n_m^{\max} \times N^{\max})$.
- (4) Fix a threshold $\epsilon > 0$.
- (5) Compute the pair (N, n_m) that is minimal (in lexicographic sense) which corresponds to a residual that is not greater than $r_{\min}$ by more than $\epsilon\%$. The use of lexicographic order enhances smaller values of N since high values of N increase the regressor dimension.

4. Description of the knowledge-based data generator

As it is explained above, the formulation of the identification problem and the proposed identification framework are validated using data issued from a detailed knowledge-based model of a beta-type Stirling engine. For a detailed discussion of the model, the reader is invited to examine the recent paper [5] where the complete derivation of the model is given. Here, only a brief description of the model derivation is given in order to underline some specific features.

This Stirling engine contains two compartments which are the expansion chamber and the compression chamber (see Fig. 3). These chambers are respectively close to the hot source with temperature T_H and the cold source with a temperature T_L . The air circulates between these two chambers through the regenerative channel. The volume of the chambers is uniquely determined by the instantaneous value of the angle θ and the fact that the lengths ℓ_c and ℓ_e are constant.

The model proposed in [5] assumes that the air obeys the ideal gas equation, which means that only two of the three variables (T, P, m) in each chamber are independent for a given value of θ (and hence the volume).

The authors of [5] choose the pair (T, m) in each chamber as the pair of independent variable leading to a state x^{th} of dimension 5 that is defined by:

$$x^{th} := (T_e \quad T_c \quad m_e \quad m_c \quad \theta)^T \quad (32)$$

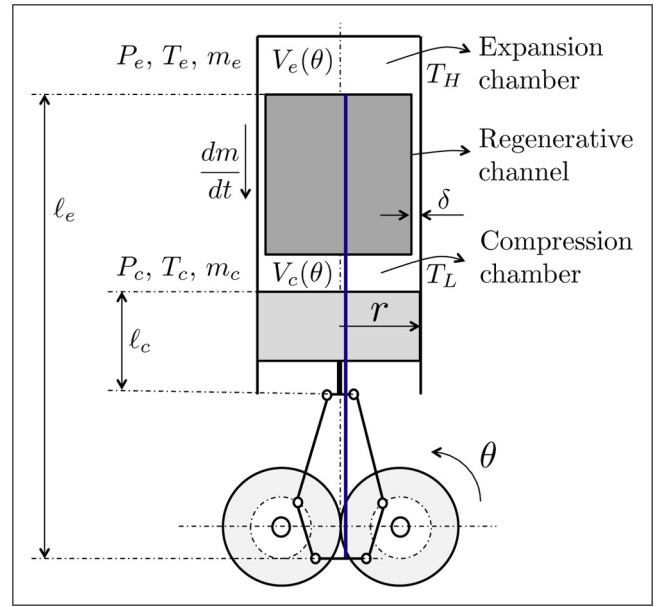


Fig. 3. Schematic view of the beta-type Stirling engine modeled in [5] and which is used in this paper to validate the proposed identification framework.

while the exogenous input vector u^{th} is given by:

$$u := (\Omega \quad T_H \quad \delta)^T \quad (33)$$

Remark 3. In operational Stirling engines, the flow rate through the regenerative channel between the expansion and the compression chambers is monitored through valves in order to control the heat transfer. In the simplified scheme of the Stirling engine considered in the present paper the variable δ seems to be constant by construction (see Fig. 3). However, once the model is obtained, one can emulate the effects of controlled flow rate on the efficiency of the engine by simulating dynamic variations of δ in order to check whether the identification structure would be able to capture the resulting dynamics.

The equations of the model take the following form:

$$\frac{dU}{dt}|_e = \dot{Q}_{in,e} - \dot{W}_{out,e} - \frac{dm}{dt} \times \left(h + \frac{v^2}{2} \right)|_e \quad (34)$$

$$\frac{dU}{dt}|_c = \dot{Q}_{in,c} - \dot{W}_{out,c} - \frac{dm}{dt} \times \left(h + \frac{v^2}{2} \right)|_c \quad (35)$$

$$\frac{dm}{dt}|_e = -\frac{dm}{dt}; \quad \frac{dm}{dt}|_c = +\frac{dm}{dt}; \quad \frac{d\theta}{dt} = \Omega \quad (36)$$

where U_e and U_c represent the internal energies of the air in each chamber, $\dot{Q}_{in,e} = (T_H - T_e)/R_{t1}$ and $\dot{Q}_{in,c} = (T_L - T_c)/R_{t2}$ are the heat exchanged by the chamber with the hot and the cold sources respectively. R_{t1} and R_{t2} are the thermal resistances of the heating head and the cooling jacket respectively. $\dot{W}_{e,out}$ and $\dot{W}_{c,out}$ are the mechanical works developed in the expansion and the compression chambers respectively. Their expressions involve the pressures P_e and P_c and the variations of the volumes V_e and V_c that depend exclusively on the current position θ and its derivative Ω .

The dm/dt is the mass flow rate through the regenerative channel (see Fig. 3). The pairs $(h, v)|_e$ and $(h, v)|_c$ represent the enthalpy and the velocity of the air at the boundaries between the chambers and the regenerative channel. The velocities v_e and v_c are linked to the mass flow rate dm/dt through the expressions:

$$v_e = \frac{|(dm)/(dt)|}{2\rho_e\pi r\delta}; \quad v_c = \frac{|(dm)/(dt)|}{2\rho_c\pi r\delta} \quad (37)$$

with r being the radius of the cylinder (see Fig. 3).

The expression of the mass flow rate $(dm)/(dt)$ is obtained by writing the equation of momentum of a fully developed annular flow of an incompressible fluid. This means that the velocity is zero at the boundaries while it depends on the radial coordinate in between (see Fig. 3). The expression involved the instantaneous pressure difference between the two chambers and the velocity in the displacer (see [5] for the detailed expressions).

Now expressing the internal energies in terms of the temperatures $U = C_v(T) \cdot T$, injecting it into Eqs. (34) and (35) and performing implicit discrete integration leads to equations of the form (in which x^+ and p^+ denote the values of x and p at the next sampling instant).

$$x^+ = f(x, u, p^+); \quad p := (P_e, P_c) \quad (38)$$

$$p = g(x) \quad (39)$$

where (39) expresses the ideal gas equation in the expansion and the compression chambers. This obviously defines a fixed-point iteration in the pressures $p = (P_e, P_c)$ that has to converge for each time step of the model integration:

$$p^{(i+1)} = g \circ f(x, u, p^{(i)}) =: G_{(x,u)}(p^{(i)}) \quad (40)$$

As it has been mentioned in [5], the convergence of the simulator needs extremely small sampling periods, our experience suggests that the sampling period:

$$\tau = 5 \times 10^{-6} \text{ s}$$

seems to be an upper bound. This is what makes the simulator quite heavy for potential integration in a whole simulation framework of the hybrid power generation station.

Fig. 4 shows a typical cycle of the beta-type Stirling engine machine described above. The cycle clearly shows how far it is from the ideal Stirling cycle which is composed of ideal isotherms and isobar curves. The cycles clearly show the positive sign of the output work as the clockwise cycle shows greater surface than the anti-clockwise cycle.

$$W_{out} := \int_0^T P_e(t) \dot{V}_e(t) dt + \int_0^T P_c(t) \dot{V}_c(t) dt \quad (41)$$

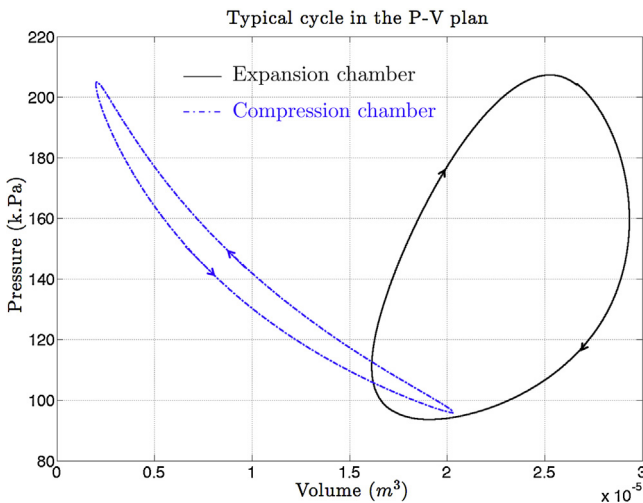


Fig. 4. Typical (P–V)-cycle of the beta-type Stirling engine used as a data generator for the validation of the proposed identification scheme.

As for the heat exchanged with the hot source which is involved in the definition of the efficiency (6), it is given by the following expression as far as this Stirling engine is concerned:

$$Q_{in} := \int_0^T \frac{T_H(t) - T_e(t)}{Rt_1} dt \quad (42)$$

which means that given the geometry of the machine that enables the volume to be tracked as a function of θ , the computation of W_{out} and Q_{in} needs the measurement of the pressures P_e and P_c as well as the temperature T_e .

5. Validation of the identification framework

The identification framework recalled in Section 3 is used here to identify two maps F_1 and F_2 such that:

$$q_1 \approx F_1(Z); \quad q_2 = F_2(Z) \quad (43)$$

where $q_1 = W_{out}$ and $q_2 = Q_{in}$ while the regressor Z is given at each instant k by (20) in which $\bar{u}$ stands for the mean value of the vector u given by (21). In the following sections, the generation of the learning data (22) and the choice of the identification parameters are shown, the identification results are given before the identified solution is validated using validation data. Finally, the computation times of the knowledge-based model and the black-box model are compared.

5.1. The learning data

The learning data is generated by simulating the knowledge-based model described in Section 4 during 40 s using the input profiles given in Fig. 5. This leads to a scenario that contains $N_{data} = 646$ cycles that are then used to build the regressor instances $Z(k)$ as well as the corresponding instances of $q_1(k) = W_{out}(k)$ and $q_2(k) = Q_{in}(k)$.

Note that the learning data is based on the following bounds on the input signals:

$$\Omega \in [40, 120](\text{rad/s}); \quad \delta \in [0.15, 0.25](\text{mm}); \quad T_H \in [600, 900](\text{K})$$

5.2. The identification parameters

The following parameters have been used in the nonlinear parameter structure:

$$n_m = 3; \quad N = 3; \quad \epsilon = 10 \quad (44)$$

which leads to a regressor length

$$n_z = N \cdot n_u = 9$$

and a total number of model parameters equal to

$$n_p = n_b + n_z = 2n_m + n_z = 15 \quad \text{parameters}$$

when reported to the number of cycles, this gives a compression factor of:

$$\text{Compression factor} = \frac{472}{15} \geq 43 \quad (45)$$

Note that the value $N = 3$ means that the prediction of the work W_{out} and the heat exchange Q_{in} during a cycle depends on the mean values of the input during the last three cycles including the current one. Recall that this feature is intimately linked to the open-loop stability of the engine as it has been explained in Section 2.

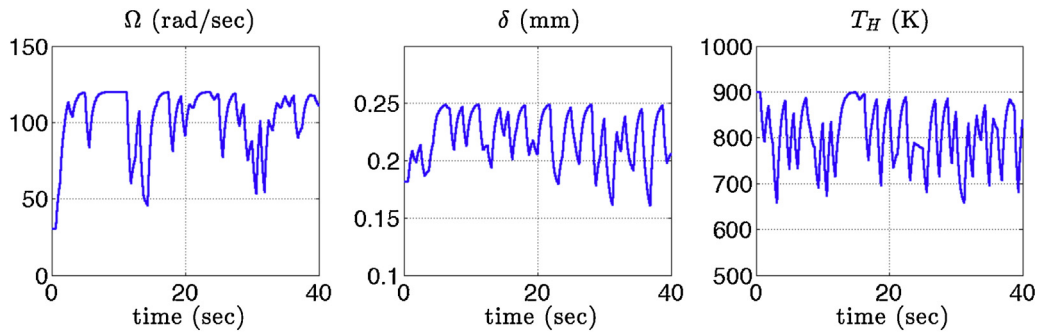


Fig. 5. The time profiles of the exogenous signals that are used to build the learning scenario.

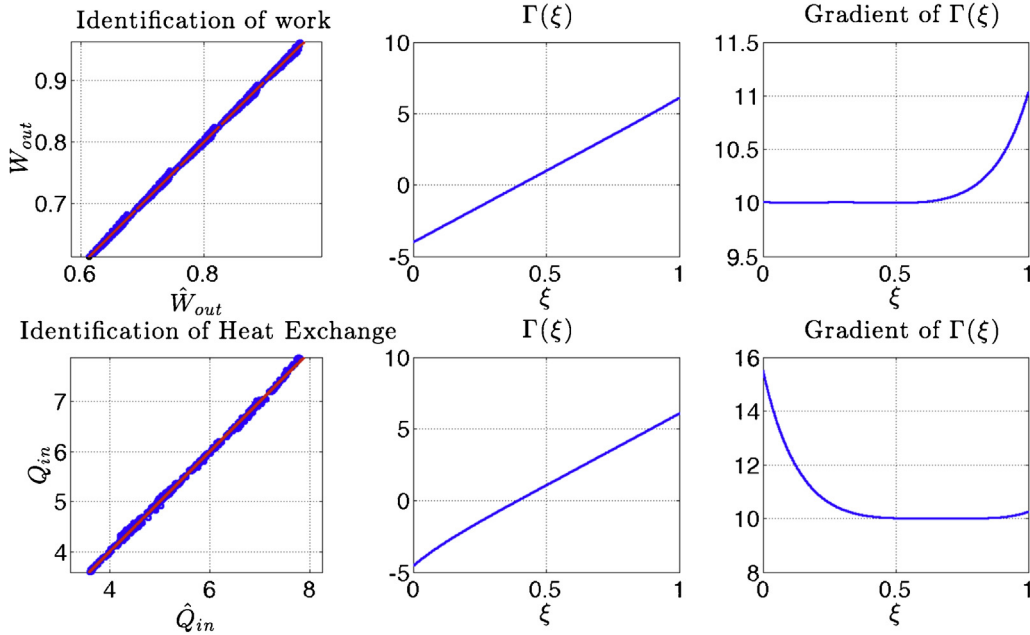


Fig. 6. Result of the identification on the learning data. Note that while the map $W_{out} = F_1(Z)$ is almost affine, higher nonlinearity is needed to approximate heat exchange $Q_{in} = F_2(Z)$.

5.3. Identification results on the learning data

The identification results are shown in Fig. 6. More precisely the top subplots show the results regarding the identification of the work while the bottom subplots show the results for the identification of the efficiency. On each row, three subplots are given: at the left, the comparison between the *true* and the identified values. At the center, the allure of the map $\Gamma(\xi)$ is given while

the righter plot shows the derivative w.r.t. ξ of the same function in order to easily appreciate the degree of nonlinearity of the function.

Fig. 7 shows a temporal comparison of the *true* and the identified quantities. Fig. 8 shows the comparison between the *true* and the identified efficiency. The latter is computed according to (6) in which the identified work $\hat{W}_{out}$ and the identified heat exchange $\hat{Q}_{in}$ are used.

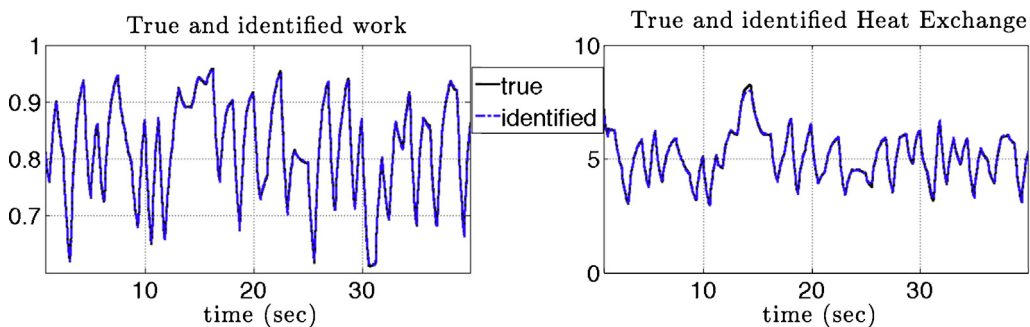


Fig. 7. Identification on the learning data: time profiles of the *true* and the identified work and heat exchange.

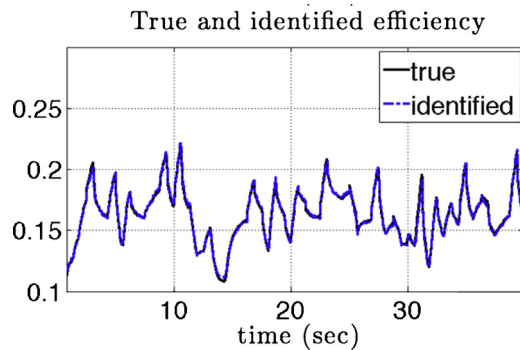


Fig. 8. Identification on the learning data: time profiles of the *true* and the identified efficiency.

5.4. Validation of the black-box model

As in any identification task, the validation of the black-box model must be done on temporal profiles of the exogenous signals that differ from the ones that are used in the learning data. Fig. 9 shows the exogenous input profiles that define the validation scenario.

The predicted profiles of the work and the heat exchange are compared to the *true* ones in Fig. 10 while the comparison between the predicted efficiency and the true one is shown in Fig. 11.

The results clearly show the quality of the black-box model as it captures the dynamics of the identified variables in a totally different scenario.

5.5. Comparison of the computation times

Table 1 shows a comparison of the computation times needed to simulate the scenario of Fig. 5 when using the knowledge-based model and the black-box model. The knowledge-based simulator

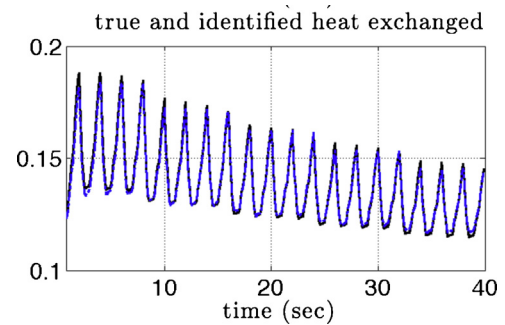


Fig. 11. Identification on the learning data: time profiles of the *true* and the predicted efficiency.

Table 1

Comparison of the computation time needed to simulate the scenario of Fig. 5 with the knowledge-based model of [5] and the black-box model. The knowledge-based model has been compiled in C language. Platform: Mac OSX, 2.3 GHz intel Core i7, 8Go 1333 MHz DDR3.

Computation times (scenario of Fig. 5)	
Black-box model	0.07 s
Knowledge-based model	39 s

has been compiled in C language in order to accelerate the computation time. The comparison suggests an acceleration factor of 570.

6. Conclusion and future work

In this contribution, a general framework for the identification of the dynamic of the key variables in a thermodynamic engine is proposed. This enables a fast simulator of the engine to be derived which can be incorporated in the simulator of the complete hybrid power generation station for simulation and/or control design and validation purposes. The data used for the identification may come

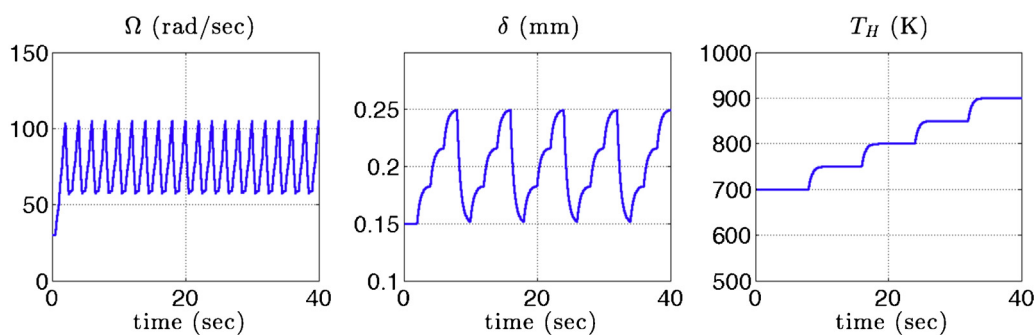


Fig. 9. Profil of the exogenous signal that is used in the validation step.

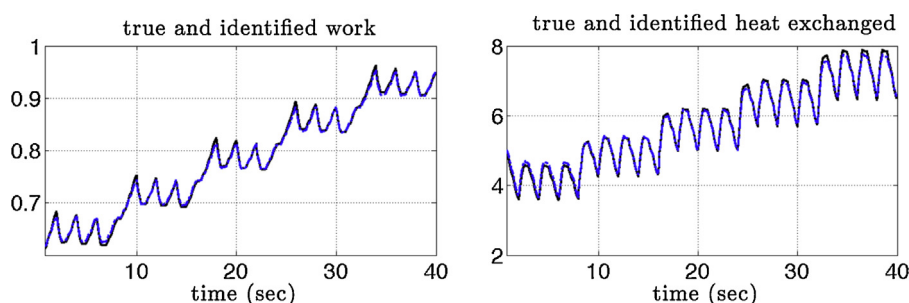


Fig. 10. validation data: time profiles of the *true* and the predicted work and heat exchange.

either from purely experimental measurement or from knowledge-based model that one would like to match in order to accelerate the simulation and the control design step.

Although the validation of the proposed solution has been shown in the case where the variables to be identified are the work and the efficiency, any other interesting variables characterizing the cycle can be identified such as the excursion of the pressure/temperature or the maximum values of these variables. The latter can be crucial in order to meet the constraints-related requirement.

Appendix A.

A.1. Definition of the basis functions $B^{(j)}(\eta)$

The basis function involved in the definition of the map Γ are given by [3]:

$$\{B^{(j)}\}_{j=1}^{n_b} := \{1\} \cup \{B_1^{(i)}\}_{i=1}^{n_m-1} \cup \{B_2^{(i)}\}_{i=1}^{n_m} \quad (\text{A.1})$$

where the number of functions in the set is equal to $n_b = 2n_m$ while the functions $B_1^{(i)}$ and $B_2^{(i)}$ are given by:

$$B_1^{(i)}(\eta) := (1 + \alpha_i) \frac{\eta}{1 + \alpha_i \eta}; \quad B_2^{(i)}(\eta) := \frac{\eta}{1 + \alpha_i(1 - \eta)} \quad (\text{A.2})$$

The coefficients α_i are given by $\alpha_i := e^{\beta(i-1)} - 1$ for some constant $\beta > 0$.

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